Journal of Engineering Physics and Thermophysics, Vol. 75, No. 6, 2002

INFLUENCE OF A VARIABLE FLUID VISCOSITY ON HEAT EXCHANGE IN LAMINAR CONVECTION

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UDC 536:24

The influence of a variable fluid viscosity on heat exchange and friction in laminar free and forced convection near the isothermal surface has been investigated numerically within a wide range of variation of Pr numbers for both the heated and cooled surfaces. The influence of the parameter $\bar{\mu}$ on the thermal and dynamic characteristics of the boundary layer has been revealed. The solutions agree well with the experimental data on the local heat exchange and the temperature field. The equations generalizing the numerical solutions have been derived to calculate heat exchange and friction in the case of a variable fluid viscosity.

Despite the numerous investigations of free-convection heat exchange in an unbounded space [1, 2], the influence of variable thermophysical properties of a liquid on the heat-exchange intensity has not been adequately studied until the present time. Therefore, investigation results are most often in poor agreement with experimental data obtained for liquids whose properties substantially depend on temperature. This is of particular importance for high-viscosity fluids, such as oils and dark petroleum products, for which the viscosity can change hundreds and thousands times within the limits of the boundary layer. The other physical properties of dropping liquids depend only slightly on temperature and virtually have no effect on heat exchange [2–4]. The influence of the temperature factor on heat exchange in laminar free convection has been studied in [3–6]; in [3–5], the solutions are obtained numerically for the case where t_w exceeds t_{fl}. In [3, 4], on the basis of theoretical and experimental investigations, it is recommended to determine the influence of a variable viscosity on heat exchange by the parameter $\bar{\nu}^{0.21} \cong \bar{\mu}^{0.21}$. Virtually similar results are obtained in [5] for $Pr_{fl} \to \infty$ and $\bar{\mu}$ = 1–50. The problem on heat exchange and resistance in the case of flow of a dropping liquid with variable physical properties past a plate has been considered in [7, 8].

In [9, 10], in generalizing experimental data, the relationship between the temperature factor and the heat exchange in free laminar and turbulent convection is considered in just the same way as in [3–5]. In most cases, the influence of variable physical properties of the fluid on heat exchange in free and forced convection is taken into account by the correction $(Pr_{fl}/Pr_w)^{0.25}$ [11] or by selection of the determining temperature which most often is taken as the average temperature of the fluid in the boundary layer [1, 2]. Although the majority of investigations is carried out for $t_w > t_{fl}$, the results of generalization are extended without any correction to the case of heat exchange where the fluid is cooled near the wall, which requires substantiation. There are virtually no scientific works on the influence of the temperature dependence of the viscosity on the heat exchange near a cooled surface ($t_w < t_{fl}$).

The differential equations of free convection in the Boussinesq approximation with account for the temperature dependence of the fluid viscosity in dimensionless form are as follows [1]:

$$\left[\frac{\mu}{\mu_{fl}} f''\right]' + 3ff'' - 2(f')^2 + \theta = 0\,, \quad \theta'' + 3\,Pr_{fl}\, f\theta' = 0\,, \quad \xi = 0: \; f = f' = 0\,; \quad \theta = 1\,;$$
$$\xi \to \infty: \; f' = 0\,; \; f'' = 0\,; \; \theta = 0\,; \; \theta' = 0\,. \tag{1}$$

When the number $Pr_{fl} \to \infty$, to solve Eq. (1) we introduce the relations [1] $\zeta = \xi\, Pr_{fl}^{1/4}$, $F = Pr_{fl}^{3/4} f$, and $H = \theta$, which transform the system of equations (1) to the form

Astrakhan State Technical University, Astrakhan, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 75, No. 6, pp. 133–136, November–December, 2002. Original article submitted June 6, 2001; revision submitted May 23, 2002.

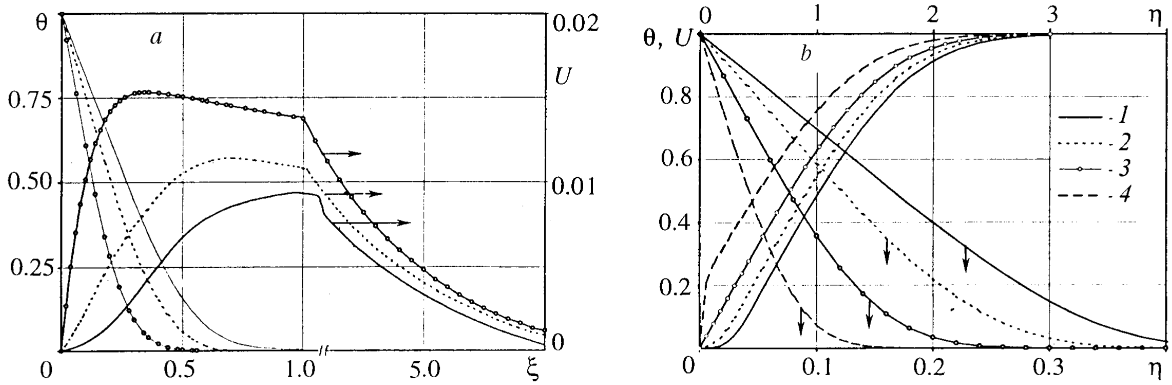

Fig. 1. Influence of a variable viscosity on the profiles of velocity and temperature in free (a) and forced (b) convection: 1) $\bar{\mu}$ = 0.01, 2) 0.1, 3) 1, and 4) 10.

$$\left[\frac{\mu}{\mu_{fl}} F''\right]' + [3FF'' - 2(F')^2]\frac{1}{\Pr_{fl}} + H = 0\,,\quad H'' + 3FH' = 0\,,\quad \zeta = 0:\;\; F = F' = 0\,,\;\; H = 1\,;$$

$$\zeta \to \infty:\;\; H = H' = F'' = 0\,. \tag{2}$$

The differential equations for forced convection near the isothermal plate in the case of a variable fluid viscosity have the form [2]

$$(\mu/\mu_{fl})\,\varphi''' + [(\mu/\mu_{fl})' + \varphi]\,\varphi'' = 0\,,$$

$$\theta'' + \Pr_{fl}\,\varphi\theta' = 0\,, \tag{3}$$

$$\eta = 0:\;\; \varphi = \varphi' = 0\,;\;\; \theta = 1\,,\;\; \eta \to \infty:\;\; \varphi' = 2\,;\;\; \varphi'' = 0\,;\;\; \theta = \theta' = 0\,.$$

The temperature dependence of the fluid viscosity was determined from the Vogel formula [3] describing rather well the relationship between the fluid viscosity and the temperature for dark petroleum products:

$$\mu/\mu_* = \exp\,[b/(t - t_*)]\,, \tag{4}$$

where μ_*, b, and t_* are constants dependent on the grade of the petroleum product and determined from two values of the viscosity at different temperatures.

With account for Eq. (4) the quantity μ/μ_{fl} in Eqs. (1)–(3) can be replaced by the relation

$$\mu/\mu_{fl} = \bar{\mu}^K\,,\quad K = \frac{(1 + T)\,\theta}{1 + T\theta}\,,\quad T = \frac{t_w - t_{fl}}{t_{fl} - t_*}\,.$$

Thus, the influence of the temperature dependence of the viscosity on the heat exchange and hydrodynamics is evaluated by two parameters: $\bar{\mu}$ and T.

The solutions of the differential equations (1), (2); (4), (5), and (6), (7) are obtained numerically according to a standard program in the MatCad system using the target method for the range of variation of the parameters $\bar{\mu}$ = 1–0.0005, T = –0.6–0, and $\Pr_{fl}$ = 10–∞ in the case of free convection and for $\Pr_{fl}$ = 10^2–10^4, $\bar{\mu}$ = 0.005–200, and T = –0.53–1 in forced convection (for example, in the case of free convection for $\Pr_{fl} \le 100$ the solutions are obtained for t_{fl} = 130°C and t_w = 30–130°C). The calculation error for $\theta(\infty)$ is less than or equal to 10^{-6}, whereas for the velocity and the velocity gradient outside the boundary layer it is $\sim 10^{-5}$.

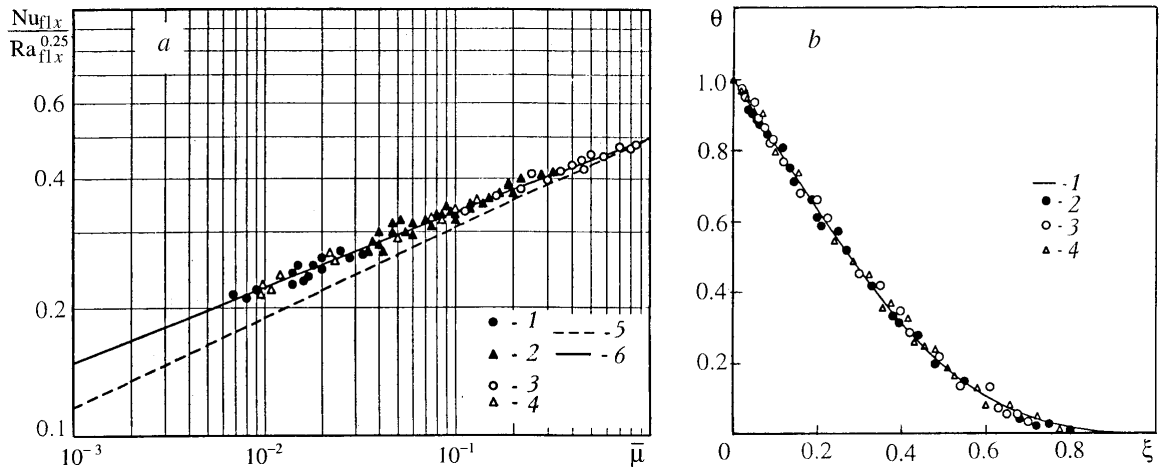

Fig. 2. Comparison of the numerical solutions with the experimental data: a) local heat exchange [1) [14]; 2) [13]; 3) oil; 4) fuel oil; 5) $Nu_{flx} = 0.497 Ra_{flx}^{1/4} \bar{\mu}^{0.21}$ [3, 4]; 6) solution, calculation from Eq. (7)]; b) temperature profile [1) solution at $\bar{\mu} = 0.1$ and $Pr_{fl} = 300$; experimental data: 2) $x = 0.135$, 3) 0.25, and 4) 0.5].

The analysis of the solutions obtained for $Pr \to \infty$ has shown that the influence of T does not exceed 1–2%, which coincides with the results of [5]. Figure 1 shows the velocity and temperature profiles in free (a) and forced (b) convection near the isothermal surface at different values of $\bar{\mu}$. From Fig. 1 it is evident that the variable viscosity of the fluid exerts a stronger influence on the dynamic parameters of the boundary layer than on the thermal parameters in both the forced and free convection. On cooling of the fluid near the wall the profiles of the velocity and the temperature are deformed toward decrease in their gradients on the wall, while on heating their gradients on the wall increase compared to the case where the physical properties are constant. The thickness of the thermal boundary layer grows on cooling and decreases on heating. The thickness of the dynamic layer depends only slightly on the parameter $\bar{\mu}$. This is explained by the fact that the thermal boundary layer for high-viscosity fluids, within the limits of which the fluid viscosity changes, is considerably thinner than the dynamic layer; therefore, the difference in the velocity profile on cooling and heating within the limits of the thermal layer has time to become smoothed over the remaining portion of the dynamic layer. As the wall temperature decreases, the maximum of the longitudinal velocity in free convection diminishes and shifts farther from the heat-exchange surface. On the velocity profile near the cooled surface, an inflection point appears, which becomes more pronounced with decrease in $\bar{\mu}$. This indicates the presence of an immobile layer near the surface. It is obvious that the model of "creeping" flow taken at large values of Pr_{fl} corresponds better to the case of heat exchange near the cooled surface than to the case of heat exchange near the heated surface. The velocity gradient on the cooled wall tending to zero with decrease in the parameter $\bar{\mu}$ and the presence of the inflection point on the velocity profile reduce the stability of the laminar flow and can lead to the separation of the boundary layer or cause the earlier transition to turbulent flow, which coincides with the analysis made in [12].

The results of the solutions allowed determination of the influence of the variable viscosity on the thermal and dynamic parameters of the boundary layer. The temperature and velocity gradients on the wall in dimensionless form in free convection with an error of no more than 0.6% are generalized by the relations

$$\theta'(0)/\theta_0'(0) = H'(0)/H_0'(0) = Nu_{flx}/Nu_{flx0} = \bar{\mu}^n , \tag{5}$$

$$F''(0)/F_0''(0) = f''(0)/f_0''(0) = \bar{\mu}^m , \tag{6}$$

where $n = 0.17$ and $m = 0.85$ for $\bar{\mu} \le 1$ ($t_w < t_{fl}$) and $n = 0.21$ and $m = 0.75$ for $\bar{\mu} \ge 1$ ($t_w > t_{fl}$).

The relative heat exchange and the friction are independent of the Prandtl number in the indicated range; this fact agrees with the results obtained in [3, 4]. The equations for the local heat exchange and the friction factor in free convection near the vertical isothermal wall with account for Eqs. (5) and (6) are as follows:

$$\mathrm{Nu}_{\mathrm{fl}x} = 0.503 \left(\frac{1}{1 + \mathrm{Pr}_{\mathrm{fl}}^{-0.5}} \right)^{0.25} \mathrm{Ra}_{\mathrm{fl}x}^{0.25} \bar{\mu}^{n} , \tag{7}$$

$$C_{fx} = 2.08 \, \mathrm{Ra}_{\mathrm{fl}x}^{-0.25} \mathrm{Pr}_{\mathrm{fl}}^{0.013} \bar{\mu}^{m-1} . \tag{8}$$

These equations hold for $\bar{\mu}$ = 0.0005–140 and $\mathrm{Pr_{fl}}$ = 10–∞.

The results of the numerical solutions are in satisfactory agreement with the experimental data obtained by the author of the present work and the data of [13, 14] on the local heat exchange (Fig. 2a) and the temperature field (Fig. 2b). The author has obtained the experimental values for the Prandtl numbers from 10^2 to $6 \cdot 10^3$ in the case of free laminar convection of high-viscosity fluids near a cooled vertical surface. The investigation technique and the description of experimental setups are given in [15].

The results of the solutions for forced convection are generalized with an error of ±1% by the following equations for the local heat exchange and the friction when $\mathrm{Pr_{fl}} \geq 10$:

$$\mathrm{Nu}_{\mathrm{fl}x} = 0.338 \, \mathrm{Re}_{\mathrm{fl}x}^{1/2} \mathrm{Pr}_{\mathrm{fl}}^{1/3} \bar{\mu}^{n} , \tag{9}$$

$$C_{fx} = 0.664 \, \mathrm{Re}_{x\mathrm{fl}}^{-0.5} \bar{\mu}^{k} , \tag{10}$$

where n = 0.18 and $k = -0.01$ for $\bar{\mu} < 1$ and n = 0.275 and $k = -0.03$ for $\bar{\mu} > 1$.

The results obtained for the friction factor agree well with those of [7] (for $\bar{\mu} < 1$, $k = -0.009$ and for $\bar{\mu} > 1$, $k = -0.035$). Also given in [7] is the equation for the local Nusselt number:

$$\mathrm{Nu}_{\mathrm{fl}x} = 0.332 \, \mathrm{Re}_{\mathrm{fl}x}^{0.5} \mathrm{Pr}_{\mathrm{fl}}^{1/3} [(\rho\mu)_{\mathrm{fl}} / (\rho\mu)_{\mathrm{w}}]^{n} , \tag{11}$$

where $n = 0.203 - 0.0291 \log [(\rho\mu)_{\mathrm{w}} / (\rho\mu)_{\mathrm{fl}}]$.

From the comparison of Eqs. (9) and (11) carried out for $\mathrm{Pr_{fl}} = 10^2$–10^4 it is evident that at $\bar{\mu}$ = 0.05–100 they are in satisfactory agreement (with an error of ±5%), but when $\bar{\mu} < 0.05$, the difference between them increases and reaches 15% at $\bar{\mu}$ = 0.01.

The conducted investigations into the influence of a variable viscosity on heat exchange and friction in laminar free and forced convection have led us to the following conclusions:

1. The influence of a variable viscosity is evaluated rather reliably by the parameter $\bar{\mu}$. The degree of action of this parameter on heat exchange and friction is different and depends on the direction of the heat flux and the type of convection, but it is self-similar relative to Pr. On heating of the fluid near the surface ($t_{\mathrm{w}} > t_{\mathrm{fl}}$), the degree of influence on the heat exchange is higher than on cooling ($t_{\mathrm{w}} < t_{\mathrm{fl}}$), whereas the degree of influence on the friction is the reverse. The influence of the variable viscosity on the heat exchange in free convection is smaller than in the case of forced convection, whereas the influence on the friction factor is the reverse.

2. On cooling of the fluid near the surface, the stability of laminar flow decreases compared to the isothermal flow. With decrease in $\bar{\mu}$ this tendency grows.

3. The calculated equations are derived for the local heat exchange and the friction in laminar free and forced convection. The results of theoretical solutions agree with experimental data and with the results obtained by other authors. The influence of the variable viscosity on the heat exchange and friction in free convection near a horizontal cylinder is the same as near a vertical surface.

NOTATION

μ and ν, dynamic and kinematic viscosities, Pa·sec and $\mathrm{m^2/sec}$; $\bar{\mu} = \mu_{\mathrm{fl}}/\mu_{\mathrm{w}}$, relative viscosity; φ and f, dimensionless stream function in forced and free convection; t, temperature, $^{\mathrm{o}}$C; x and y, longitudinal and transverse coordinates, m; $\eta = (y/2x)\mathrm{Re}_{\mathrm{fl}x}^{1/2}$ and $\xi = (y/x)(\mathrm{Gr}_{\mathrm{fl}x}/4)^{1/4}$, similarity variables in forced and free convection; $H = \theta =$

$(t_{fl} - t)/(t_{fl} - t_w)$, dimensionless temperature; $\varphi(\eta)$ and $f(\xi)$, stream function in forced and free convection, respectively; Nu, Ra, Re, and Pr, Nusselt, Rayleigh, Reynolds, and Prandtl similarity numbers, respectively; C_f, friction factor on the wall. Subscripts: fl, fluid; w, wall; x, local value; 0, for constant physical properties of the fluid.

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